28.10. The hole that the conducting wire passes through is so small that it can be ignored.
(a) Positive charge has been transferred to the small sphere, so it has a positive charge.
(b) Negative. A Gaussian surface through the conductor of the larger sphere must contain a net charge of zero, so the inner surface of the larger conductor is negatively charged. The magnitude of the charge is the same as that of the positive charge placed on the smaller sphere.
(c) The negative charge on the inner surface of the larger sphere came from its outer surface (all excess charge lies on the surface of a conductor) so the outside of the larger sphere has a positive charge. This can also be seen by considering a Gaussian surface outside the larger sphere. It contains a net positive charge (that on the smaller sphere, since the larger sphere is neutral). Since the charge on the inner surface is equal and opposite to that on the smaller sphere, the outside of the larger sphere must be positively charged.

28.4. Model: The electric flux "flows" *out* of a closed surface around a region of space containing a net positive charge and *into* a closed surface surrounding a net negative charge.
Visualize: Please refer to Figure EX28.4. Let A be the area in m^2 of each of the six faces of the cube.
Solve: The electric flux is defined as $\Phi_e = \vec{E} \cdot \vec{A} = EA\cos\theta$, where θ is the angle between the electric field and a line *perpendicular* to the plane of the surface. The electric flux *out* of the closed cube surface is

$$\Phi_{out} = (20 \text{ N/C} + 20 \text{ N/C} + 10 \text{ N/C})A\cos 0° = (50A) \text{ N m}^2\text{/C}$$

Similarly, the electric flux *into* the closed cube surface is

$$\Phi_{in} = (15 \text{ N/C} + 15 \text{ N/C} + 15 \text{ N/C})A\cos 180° = -(45A) \text{ N m}^2\text{/C}$$

The net electric flux is $(50A)\text{ N m}^2\text{/C} - (45A)\text{ N m}^2\text{/C} = (5A)\text{ N m}^2\text{/C}$. Since the net electric flux is positive (i.e., outward), the closed box contains a positive charge.

28.12. Model: The electric field is uniform over the rectangle in the *xy* plane.
Solve: **(a)** The area vector is perpendicular to the *xy* plane. Thus

$$\vec{A} = (2.0 \text{ cm} \times 3.0 \text{ cm})\hat{k} = (6.0 \times 10^{-4} \text{ m}^2)\hat{k}$$

The electric flux through the rectangle is

$$\Phi_e = \vec{E} \cdot \vec{A} = (50\hat{i} + 100\hat{k}) \cdot (6.0 \times 10^{-4}\hat{k}) \text{ N m}^2\text{/C} = 6.0 \times 10^{-2} \text{ N m}^2\text{/C}$$

(b) The electric flux is

$$\Phi_e = \vec{E} \cdot \vec{A} = (50\hat{i} + 100\hat{j}) \cdot (6.0 \times 10^{-4}\hat{k}) \text{ N m}^2\text{/C} = 0 \text{ N m}^2\text{/C}$$

Assess: In (b), $\vec{E}$ is in the plane of the rectangle. That is why the flux is zero.

28.19. Visualize: Please refer to Figure EX28.19.
Solve: For *any* closed surface that encloses a total charge Q_{in}, the net electric flux through the surface is $\Phi_e = Q_{in}/\varepsilon_0$. We can write three equations from the three closed surfaces in the figure:

$$\Phi_A = -\frac{q}{\varepsilon_0} = \frac{q_1+q_3}{\varepsilon_0} \Rightarrow q_1 + q_3 = -q \qquad \Phi_B = \frac{3q}{\varepsilon_0} = \frac{q_1+q_2}{\varepsilon_0} \Rightarrow q_1 + q_2 = 3q$$

$$\Phi_C = \frac{-2q}{\varepsilon_0} = \frac{q_2+q_3}{\varepsilon_0} \Rightarrow q_2 + q_3 = -2q$$

Subtracting third equation from the first,

$$q_1 - q_2 = +q$$

Adding second equation to this equation,

$$2q_1 = +4q \Rightarrow q_1 = 2q$$

That is, $q_1 = +2q$, $q_2 = +q$, and $q_3 = -3q$.

28.25. Model: The excess charge on a conductor resides on the outer surface.
Solve: The electric field at the surface of a charged conductor is

$$\vec{E}_{\text{surface}} = \left(\frac{\eta}{\varepsilon_0},\ \text{perpendicular to surface}\right)$$

$$\Rightarrow \eta = \varepsilon_0 E_{\text{surface}} = \left(8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2\right)\left(3.0\times10^{6}\ \text{N/C}\right) = 2.7\times10^{-5}\ \text{C/m}^2$$

Assess: It is the air molecules just above the surface that "break down" when the E-field becomes strong enough to accelerate stray charges to approximately 15 eV between collisions, thus causing collisional ionization. It does not make any difference whether E points toward or away from the surface.

28.30. Model: The electric field over the five surfaces is uniform.
Visualize: Please refer to Figure P28.30.
Solve: The electric flux through a surface area $\vec{A}$ is $\Phi_e = \vec{E}\cdot\vec{A} = EA\cos\theta$ where θ is the angle between the electric field and a line perpendicular to the plane of the surface. The electric field is perpendicular to side 1 and is parallel to sides 2, 3, and 5. Also the angle between $\vec{E}$ and $\vec{A}_4$ is 60°. The electric fluxes through these five surfaces are

$$\Phi_1 = E_1A_1\cos\theta_1 = (400\ \text{N/C})(2\ \text{m} \times 4\ \text{m})\cos(180°) = -3200\ \text{N m}^2/\text{C}$$
$$\Phi_2 = E_2A_2\cos90° = \Phi_3 = \Phi_5 = 0\ \text{N m}^2/\text{C}$$
$$\Phi_4 = E_4A_4\cos\theta_4 = (400\ \text{N/C})\big((2\ \text{m}/\sin30°)\times 4\ \text{m}\big)\cos60° = +6400\ \text{N m}^2/\text{C}$$

Assess: Because the flux into these five faces is equal to the flux out of the five faces, the net flux is zero, as we found.

28.34. Solve: For *any* closed surface that encloses a total charge Q_{in}, the net electric flux through the closed surface is $\Phi_e = Q_{in}/\varepsilon_0$. The total flux through the cube is

$$\Phi_e = 6\left(100\ \text{N m}^2/\text{C}\right) = \frac{Q_{in}}{\varepsilon_0} \Rightarrow Q_{in} = \left(600\ \text{N m}^2/\text{C}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N m}^2\right) = 5.31\times10^{-9}\ \text{C} = 5.31\ \text{nC}$$

28.40. Model: The excess charge on a conductor resides on the outer surface. The charge distribution on the two spheres is assumed to have spherical symmetry.

Visualize: Please refer to Figure P28.40. The Gaussian surfaces with radii r = 8 cm, 10 cm, and 17 cm match the symmetry of the charge distribution. So, $\vec{E}$ is perpendicular to these Gaussian surfaces and the field strength has the same value at all points on the Gaussian surface.

Solve: **(a)** Gauss's law is $\Phi_e = \oint \vec{E} \cdot d\vec{A} = Q_{in}/\varepsilon_0$. Applying it to a Gaussian surface of radius 8 cm,

$$Q_{in} = -\varepsilon_0 E A_{sphere} = -\left(8.85\times10^{-12}\ \text{C}^2/\text{N m}^2\right)\left(15{,}000\ \text{N/C}\right)\left[4\pi\left(0.08\ \text{m}\right)^2\right] = -1.07\times10^{-8}\ \text{C}$$

Because the excess charge on a conductor resides on its outer surface and because we have a solid metal sphere inside our Gaussian surface, Q_{in} is the charge that is located on the exterior surface of the inner sphere.

(b) In electrostatics, the electric field within a conductor is zero. Applying Gauss's law to a Gaussian surface just inside the inside surface of the hollow sphere at r = 10 cm,

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \Rightarrow Q_{in} = 0\ \text{C}$$

That is, there is no net charge. Because the inner sphere has a charge of -1.07×10^{-8} C, the inside surface of the hollow sphere must have a charge of $+1.07 \times 10^{-8}$ C.

(c) Applying Gauss's law to a Gaussian surface at r = 17 cm,

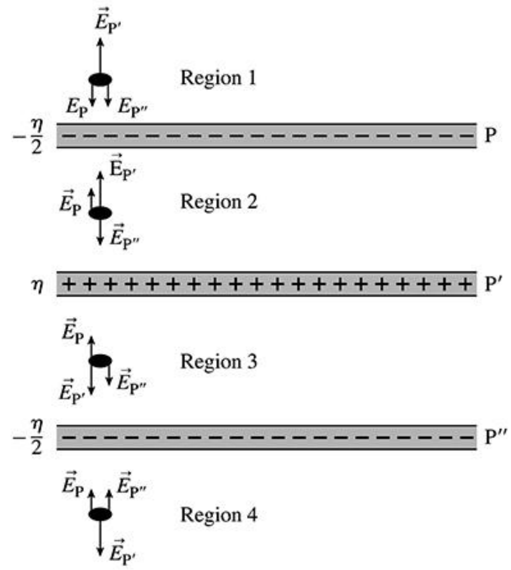
$$Q_{in} = \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 E A_{sphere} = \left(8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2\right)\left(15{,}000\ \text{N/C}\right)4\pi\left(0.17\ \text{m}\right)^2 = 4.82\times10^{-8}\ \text{C}$$

This value includes the charge on the inner sphere, the charge on the inside surface of the hollow sphere, and the charge on the exterior surface of the hollow sphere due to polarization. Thus,

$$Q_{\text{exterior hollow}} + \left(1.07\times10^{-8}\ \text{C}\right) + \left(-1.07\times10^{-8}\ \text{C}\right) = 4.82\times10^{-8}\ \text{C}$$

$$\Rightarrow Q_{\text{exterior hollow}} = 4.82\times10^{-8}\ \text{C}$$

28.47. Model: The three planes of charge are infinite planes.
Visualize:



From planar symmetry the electric field can point straight toward or away from the plane. The three planes are labeled as P (top), P′, and P″(bottom).

Solve: From Example 28.6, the electric field of an infinite charged plane of charge density η is

$$E_{\text{plane}} = \frac{\eta}{2\varepsilon_0} \Rightarrow E_{\text{P}} = E_{\text{P}''} = \frac{\eta}{4\varepsilon_0} = \frac{E_{\text{P}'}}{2}$$

In region 1 the three electric fields are

$$\vec{E}_{\text{P}} = -\frac{\eta}{4\varepsilon_0}\hat{j} \qquad \vec{E}_{\text{P}'} = \frac{\eta}{2\varepsilon_0}\hat{j} \qquad \vec{E}_{\text{P}''} = -\frac{\eta}{4\varepsilon_0}\hat{j}$$

Adding the three contributions, we get $\vec{E}_{\text{net}} = \vec{0}$ N/C.

In region 2 the three electric fields are

$$\vec{E}_{\text{P}} = \frac{\eta}{4\varepsilon_0}\hat{j} \qquad \vec{E}_{\text{P}'} = \frac{\eta}{2\varepsilon_0}\hat{j} \qquad \vec{E}_{\text{P}''} = \frac{-\eta}{4\varepsilon_0}\hat{j}$$

Thus, $\vec{E}_{\text{net}} = (\eta/2\varepsilon_0)\hat{j}$.

In region 3,

$$\vec{E}_{\text{P}} = \frac{\eta}{4\varepsilon_0}\hat{j} \qquad \vec{E}_{\text{P}'} = -\frac{\eta}{2\varepsilon_0}\hat{j} \qquad \vec{E}_{\text{P}''} = -\frac{\eta}{4\varepsilon_0}\hat{j}$$

Thus, $\vec{E}_{\text{net}} = -(\eta/2\varepsilon_0)\hat{j}$.

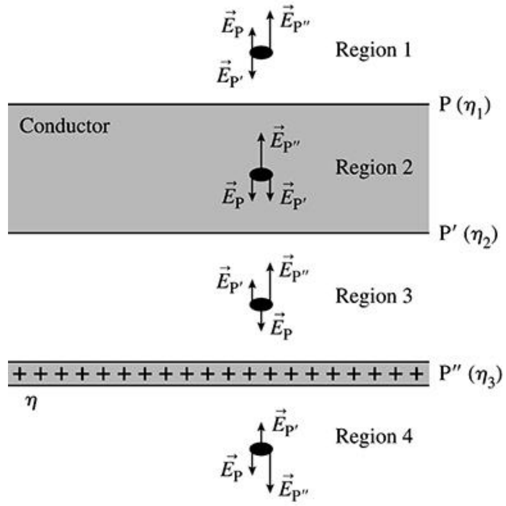
In region 4,

$$\vec{E}_{\text{P}} = \frac{\eta}{4\varepsilon_0}\hat{j} \qquad \vec{E}_{\text{P}'} = -\frac{\eta}{2\varepsilon_0}\hat{j} \qquad \vec{E}_{\text{P}''} = \frac{\eta}{4\varepsilon_0}\hat{j}$$

Thus $\vec{E}_{\text{net}} = \vec{0}$ N/C.

28.49. Model: The infinitely wide plane of charge with surface charge density η polarizes the infinitely wide conductor.

Visualize:



Because $\vec{E} = \vec{0}$ in the metal there will be an induced charge polarization. The face of the conductor adjacent to the plane of charge is negatively charged. This makes the other face of the conductor positively charged. We thus have three infinite planes of charge. These are P (top conducting face), P′ (bottom conducting face), and P″(plane of charge).

Solve: Let η_1, η_2, and η_3 be the surface charge densities of the three surfaces with η_2 a negative number. The electric field due to a plane of charge with surface charge density η is $E = \eta/2\varepsilon_0$. Because the electric field inside a conductor is zero (region 2),

$$\vec{E}_{P} + \vec{E}_{P'} + \vec{E}_{P''} = \vec{0}\text{ N/C} \Rightarrow -\frac{\eta_1}{2\varepsilon_0}\hat{j} + \frac{\eta_2}{2\varepsilon_0}\hat{j} + \frac{\eta_3}{2\varepsilon_0}\hat{j} = \vec{0}\text{ N/C} \Rightarrow -\eta_1 + \eta_2 + \eta = 0\text{ C/m}^2$$

We have made the substitution $\eta_3 = \eta$. Also note that the field inside the conductor is downward from planes P and P′ and upward from P″. Because $\eta_1 + \eta_2 = 0\text{ C/m}^2$, because the conductor is neutral, $\eta_2 = -\eta_1$. The above equation becomes

$$-\eta_1 - \eta_1 + \eta = 0\text{ C/m}^2 \Rightarrow \eta_1 = \tfrac{1}{2}\eta \Rightarrow \eta_2 = -\tfrac{1}{2}\eta$$

We are now in a position to find electric field in regions 1–4.

For region 1,

$$\vec{E}_{P} = \frac{\eta}{4\varepsilon_0}\hat{j} \qquad \vec{E}_{P'} = -\frac{\eta}{4\varepsilon_0}\hat{j} \qquad \vec{E}_{P''} = \frac{\eta}{2\varepsilon_0}\hat{j}$$

The electric field is $\vec{E}_{\text{net}} = \vec{E}_{P} + \vec{E}_{P'} + \vec{E}_{P''} = (\eta/2\varepsilon_0)\hat{j}$.

In region 2, $\vec{E}_{\text{net}} = \vec{0}$ N/C. In region 3,

$$\vec{E}_{P} = -\frac{\eta}{4\varepsilon_0}\hat{j} \qquad \vec{E}_{P'} = \frac{\eta}{4\varepsilon_0}\hat{j} \qquad \vec{E}_{P''} = \frac{\eta}{2\varepsilon_0}\hat{j}$$

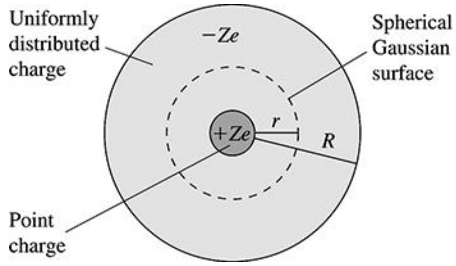
The electric field is $\vec{E}_{\text{net}} = (\eta/2\varepsilon_0)\hat{j}$.

In region 4,

$$\vec{E}_{P} = -\frac{\eta}{4\varepsilon_0}\hat{j} \qquad \vec{E}_{P'} = \frac{\eta}{4\varepsilon_0}\hat{j} \qquad \vec{E}_{P''} = -\frac{\eta}{2\varepsilon_0}\hat{j}$$

The electric field is $\vec{E}_{\text{net}} = -(\eta/2\varepsilon_0)\hat{j}$.

28.54. Model: Assume that the negative charge uniformly distributed in the atom has spherical symmetry.
Visualize:



The nucleus is a positive point charge $+Ze$ at the center of a sphere of radius R. The spherical symmetry of the charge distribution tells us that the electric field must be radial. We choose a spherical Gaussian surface to match the spherical symmetry of the charge distribution and the field. The Gaussian surface is at $r < R$, which means that we will calculate the amount of charge contained in this surface.

Solve: (a) Gauss's law is $\oint \vec{E}\cdot d\vec{A} = Q_{\text{in}}/\varepsilon_0$. The amount of charge inside is

$$Q_{\text{in}} = \rho\left(\frac{4\pi}{3}r^3\right) + Ze = \frac{(-Ze)}{\left(\frac{4\pi}{3}R^3\right)}\left(\tfrac{4\pi}{3}r^3\right) + Ze = -(Ze)\frac{r^3}{R^3} + Ze = Ze\left[1-\frac{r^3}{R^3}\right]$$

$$\Rightarrow E_{\text{in}}\left(4\pi r^2\right) = \frac{Ze}{\varepsilon_0}\left[1-\frac{r^3}{R^3}\right] \Rightarrow E_{\text{in}} = \frac{Ze}{4\pi\varepsilon_0}\left[\frac{1}{r^2}-\frac{r}{R^3}\right]$$

(b) At the surface of the atom, $r = R$. Thus,

$$E_{\text{in}} = \frac{Ze}{4\pi\varepsilon_0}\left[\frac{1}{R^2}-\frac{R}{R^3}\right] = 0 \text{ N/C}$$

This is an expected result, which can be quickly obtained from Gauss's law. Applying Gauss's law to a Gaussian surface just outside $r = R$. Because the atom is electrically neutral, $Q_{\text{in}} = 0$. Thus

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0} = 0 \Rightarrow E = 0 \text{ N/C}$$

(c) The electric field strength at $r = \frac{1}{2}R = 0.050\text{ nm}$ is

$$E_{\text{in}} = 92\left(1.60\times10^{-19}\text{ C}\right)\left(9.0\times10^{9}\text{ C}^2/\text{Nm}^2\right)\left[\frac{1}{(0.050\text{ nm})^2}-\frac{(0.050\text{ nm})}{(0.10\text{ nm})^3}\right] = 4.6\times10^{13}\text{ N/C}$$

28.6. $\Phi_A = +4q/\varepsilon_o$ $\Phi_B = -4q/\varepsilon_o$ $\Phi_C = 0$ $\Phi_D = +3q/\varepsilon_o$ $\Phi_E = 0$